# Where to Deploy Reconfigurable Intelligent Surfaces in the Presence of Blockages?

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***Abstract*—Wireless communications aided by reconfigurable intelligent surfaces (RISs) is a promising way to improve the coverage for cellular users. The controlled reflection of the signal from RISs is especially useful in mm-wave networks when the direct link between a cellular user and its serving base station (BS) is weak or unavailable due to blockages. But the joint blockage of the user-RIS and the user-BS links may significantly degrade the performance of RIS-aided transmissions. This paper aims to study the effect of joint blockages on downlink performance. When the RIS locations are coupled with BS locations, using tools from stochastic geometry, we obtain an optimal placement of RISs either to minimize the joint blockage probability of the user-RIS and the user-BS links or to maximize the downlink coverage probability. The results show that installing RISs on the street intersections improves the coverage probability. For users associated with BSs that are deployed sufficiently close to intersections, the intersection-mounted RISs offer a better coverage performance compared to BS-coupled RISs.**

## I. Introduction

Seamless connectivity, ubiquitous coverage, and high-speed data rates drive the search for new physical layer technologies [1], [2]. A promising step towards achieving these goals is the use of RISs, realized either by composite meta-materials or by close packing of passive antenna elements [3]. An RIS can be programmed to function as a transmitting [3] or a receiving node [4]. It can also be optimized to operate as an anomalous reflector [5]; in this mode, the reflected wave from the RIS can be steered to either enhance signal coverage or localize and track cellular users. In this paper, we focus on RISs acting as intelligent reflectors and investigate their placement in a millimeter wave (mm-wave) cellular network to optimize the downlink signal-to-noise ratio (SNR) coverage.

Since the use of RISs for wireless communications is relatively new, the focus has been restricted to the understanding of its key features and properties in simple networks consisting of a few BSs, users, and RISs [6]. In a large-scale network, the work in [7] has developed a stochastic geometry-based model to evaluate RIS-aided multi-cell networks. The deployment of RISs with respect to the BS and the user dictates the end-to-end (i.e., BS to the user via the RIS) path loss, which in turn, decides the communication performance [8]. In particular, the end-to-end path loss varies as the product of the path losses of the individual (i.e., BS to the RIS and the RIS to the user) links [9]. Accordingly, it is minimized when the RIS is installed in close proximity to the BS or the user. But such a deployment strategy increases the chance that the BS and the RIS are simultaneously blocked from the perspective of the user, leading to the outage. Since blockages are an integral part of a dense urban environment and significantly affect mm-wave communications, it is important to include their impact in the analysis [10], [11]. The impact of correlation in the blocking event of co-existing links, due to either static or dynamic objects, is generally ignored while characterizing and dimensioning mm-wave networks. This inhibits the derivation of realistic system design insights for mm-wave cellular networks. A recent work has characterized the joint blockage probability of two co-existing links in a 2D mm-wave cellular network using random shape theory [12] and studied its impact on the coverage probability [13]. However, past works have not studied the impact of link blockages on the performance of systems with RISs, including the scenarios when there is correlation between the blockage of the links from a BS and an RIS to the user, which is the main focus of this paper.

This paper makes the following contributions:

1) We study the impact of blockages on the deployment planning of RISs in an urban setting.
2) When the installation of RISs is coupled with BS locations along the streets, using tools from stochastic geometry [14], we obtain a closed-form expression of the probability that the links from the BS and the coupled RIS to the user are jointly blocked.
3) We provide an analytical expression of the coverage probability under such joint blockages of links with and without intersection-mounted RISs.
4) We numerically obtain the optimal distance of the RIS from the BS and its height that minimize the joint blockage probability and maximize the coverage probability.
5) We consider the intersections of streets (an important scenario in an urban setting), where we calculate the coverage probability for the typical intersection user and compare its performance with that of the typical general user.

**Notation:** $\bar{\mathsf{E}}$ denotes the complementary event of an event $\mathsf{E}$. $h$ and $r$ denote vertical and horizontal distances, while the Euclidean distance is denoted by $d$. The typical user, its serving BS, the associated RIS are generally denoted by $\mathrm{u}$, $\mathrm{b}$, and $\mathrm{s}$. The intersection user and RIS are denoted by $\mathrm{i}$ and $\mathrm{x}$.

## II. System Model

Consider an urban wireless propagation environment that consists of densely located blockages, e.g., buildings, trees, and vehicles. For this scenario, we study a downlink cellular network consisting of BSs aided by RISs. The BSs are deployed along the streets (e.g., on the lamp posts) and operate

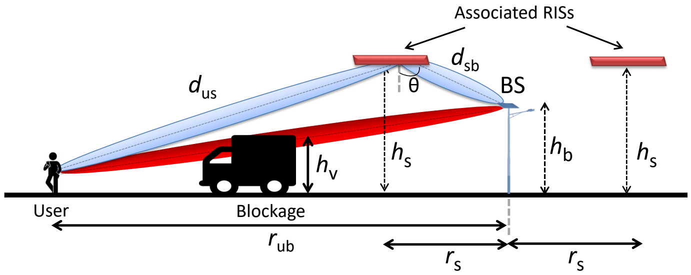


Fig. 1. Illustration of the system model under consideration.

in an mm-wave band. For each BS, two RISs are installed on either side of the BS to augment the downlink transmissions.

**Network geometry.** The street system is modeled by a Poisson line process (PLP) $\Phi_{\mathrm{r}}$ with intensity $\lambda_{\mathrm{r}}$ defined on a half-cylinder $[0, 2\pi) \times \mathbb{R}^{+}$. The BSs, each of height $h_{\mathrm{b}}$, are deployed on the tessellation of the streets. Their locations are modeled as the points of a one-dimensional Poisson point process (PPP) $\Phi_{\mathrm{b}k}$ on each street $l_k \in \Phi_{\mathrm{r}}$, with intensity $\lambda_{\mathrm{b}} \in \mathbb{R}^{+}$. Here, $k \in \mathbb{N}$ is the index of the street. Let $\Phi_{\mathrm{b}} := \cup_k \Phi_{\mathrm{b}k}$ denote the overall BS point process, which by construction, is a Poisson line Cox process (PLCP). The direct link between any two points located on different streets $l_k$ and $l_j$, $k \neq j$, is assumed to be blocked by buildings.

The RIS placement is coupled with BS locations. Specifically, two RISs with $N$ elements (or meta-atoms) with dimensions $a \times b$ are deployed in conjunction with a BS at a distance $r_{\mathrm{s}}$ from the BS on each side of it, and at a height of $h_{\mathrm{s}}$. We assume that $h_{\mathrm{s}}$ is always greater than $h_{\mathrm{b}}$. The typical (general) user is randomly located at the typical street.

**Blockages.** The link between a user and a BS and that between a user and an RIS may be blocked. Let $h_{\mathrm{v}}$ denote the height of a blockage. On each street $l_i \in \Phi_{\mathrm{r}}$, the locations of blockages follow a one-dimensional PPP $\Phi_{\mathrm{v}i}$, which is independent of $\Phi_{\mathrm{b}i}$. In the case of a non line-of-sight (NLOS) link (due to blockages present), we assume that the received power at the user is negligible [15].[1]

**User association.** Assume that the typical user connects to the nearest BS on the same street. The probability density function (PDF) of the horizontal distance between the typical user and the base of the serving BS is given by [14]

$$f_{r_{\mathrm{ub}}}(x) = 2\lambda_{\mathrm{b}} \exp\left(-2\lambda_{\mathrm{b}} x\right), \quad x \geq 0. \tag{1}$$

A BS serves the user via the direct link if it is in the line-of-sight (LOS) state. If the direct link is blocked (i.e., it is in the NLOS state), the BS uses the reflected link from the associated RIS by beamforming towards the RIS. The transmission from the BS to the user experiences an outage if both the links from the BS and the associated RISs to the user are blocked.

**Pathloss.** For an LOS link, the instantaneous received power at a distance $d$ is $S_{\mathrm{r}} = P_{\mathrm{t}} K g_{\mathrm{tr}} G_{\mathrm{t}} G_{\mathrm{r}} d^{-\alpha}$, where $P_{\mathrm{t}}$ is the transmit power of a BS, $G_{\mathrm{t}}$ and $G_{\mathrm{r}}$ are the transmit and receive beamforming gains, respectively. $K$ and $\alpha$ are the path-loss constant and the path-loss exponent, respectively. Due to low local scattering in mm-wave, we assume Nakagami fading, i.e., the fading power $g_{\mathrm{tr}}$ is Gamma distributed with parameter $n_0$.

**SNR model.** Let $\beta \triangleq K\frac{G_{\mathrm{t}} G_{\mathrm{r}}}{(4\pi)^2}$. When there exists an LOS path between the serving BS and the typical user, the user downlink SNR is

$$S = \frac{P_{\mathrm{t}}}{N_0} g_{\mathrm{bu}} \beta d_{\mathrm{ub}}^{-\alpha}, \tag{2}$$

where $d_{\mathrm{ub}} = \sqrt{r_{\mathrm{ub}}^2 + h_{\mathrm{b}}^2}$ is the distance of the BS to the user and $g_{\mathrm{bu}}$ denotes the fading power gain on the link between the user and the BS. $N_0$ is the noise power. If the link from the typical user to the BS is blocked and the RIS is in LOS, the SNR with the reflected signal at the user is given as

$$S' = \frac{P_{\mathrm{t}}}{N_0} N^2 \beta \left(d_{\mathrm{us}} d_{\mathrm{sb}}\right)^{-\alpha} \cos^2(\theta) g_{\mathrm{su}}, \tag{3}$$

where $d_{\mathrm{us}}$ is the distance of the user from the RIS, $d_{\mathrm{sb}}$ is the distance of the RIS from the BS, $g_{\mathrm{su}}$ is the fading power gain on the link between the RIS and the user, and $\theta$ is the incidence angle at RIS. Naturally, we have $d_{\mathrm{sb}} = \sqrt{r_{\mathrm{s}}^2 + (h_{\mathrm{s}} - h_{\mathrm{b}})^2}$, $d_{\mathrm{us}} = \sqrt{(r_{\mathrm{ub}} - r_{\mathrm{s}})^2 + h_{\mathrm{s}}^2}$, and $\theta = \arctan\left((h_{\mathrm{s}} - h_{\mathrm{b}})/r_{\mathrm{s}}\right)$.

## III. Characterization of Link Blockages

The link between the serving BS and the typical user is blocked if a blockage is located within a distance of $r_{\mathrm{ub}} \frac{h_{\mathrm{v}}}{h_{\mathrm{b}}}$ from the user towards the BS. Let $\mathsf{E}_{\mathrm{b}}$ and $\mathsf{E}_{\mathrm{s}}$ denote the events that the links from the user to the BS and the RIS are blocked, respectively. Since the locations of the blockages are modeled as PPP, we have the following result.

**Lemma 1.** *The probability that the link between the typical general user and the serving BS is blocked conditioned on horizontal distance $r_{\mathrm{ub}}$ between them is given by [14]*

$$\mathbb{P}(\mathsf{E}_{\mathrm{b}} \mid r_{\mathrm{ub}}) = 1 - \exp\left(-\lambda_{\mathrm{v}} r_{\mathrm{ub}} h_{\mathrm{v}} / h_{\mathrm{b}}\right). \tag{4}$$

Recall that the BS switches its beam towards the RIS if its direct link to the user is blocked. Thus, the user is in a complete signal outage when the direct and the RIS links are simultaneously blocked.

**Lemma 2.** *Conditioned on the horizontal distance $r_{\mathrm{ub}}$ between the typical user and the BS, the joint probability of the events $\mathsf{E}_{\mathrm{s}}$ and $\mathsf{E}_{\mathrm{b}}$ is given by*

$$\mathbb{P}(\mathsf{E}_{\mathrm{b}}, \mathsf{E}_{\mathrm{s}} \mid r_{\mathrm{ub}}) = \begin{cases} \left(1 - \exp\left(-\lambda_{\mathrm{v}} \frac{h_{\mathrm{v}}}{h_{\mathrm{b}}} r_{\mathrm{ub}}\right)\right) & \\ \quad \times \left(1 - \exp\left(-\lambda_{\mathrm{v}} (r_{\mathrm{s}} - r_{\mathrm{ub}}) \frac{h_{\mathrm{v}}}{h_{\mathrm{s}}}\right)\right) & \text{if } 0 \leq r_{\mathrm{ub}} \leq r_{\mathrm{s}}, \\ 1 - \exp\left(-\lambda_{\mathrm{v}} (r_{\mathrm{ub}} - r_{\mathrm{s}}) \frac{h_{\mathrm{v}}}{h_{\mathrm{s}}}\right) & \text{if } r_{\mathrm{ub}} > r_{\mathrm{s}}. \end{cases}$$

*Proof:* For $0 \leq r_{\mathrm{ub}} \leq r_{\mathrm{s}}$, the typical user is located between the serving BS and the RIS. Accordingly, the joint blockage probability is the product of the blockage probabilities of individual links from the user to the BS and the RIS. On the contrary, for $r_{\mathrm{ub}} \geq r_{\mathrm{s}}$, the user-BS and the user-RIS links are jointly blocked when a blockage is located at a distance of $r_{\mathrm{ub}} - r_{\mathrm{s}}$ from the user. The result follows from the void probability of the PPP [16]. ■

[1] Note that our model can be extended to incorporate the impact of non-negligible NLOS signals.

**Remark 1.** *Lemma 2 shows that if an RIS is placed very close or very far from the BS, the joint blockage probability is high. For close deployments of the RIS to the BS, the user-BS link is blocked whenever the user-RIS link is blocked. On the contrary, when an RIS is deployed far from the BS, the individual links from the user to the BS and the RIS have a high probability of blockage. In particular, for a given $r_{\rm ub}$, the joint blockage probability is minimum when $r_{\rm s} = r_{\rm ub}$. This means that from a particular user's perspective, the RIS should be located directly above the user. Since there are many users, one should select a location of the RIS that results in the smallest blockage probability on an average.*

By unconditioning over $r_{\rm bu}$, we obtain the average joint blockage probability as $\mathcal{B}_{\rm u} = \mathbb{P}(\mathsf{E}_{\rm b}, \mathsf{E}_{\rm s}) = \int_0^\infty \mathbb{P}(\mathsf{E}_{\rm b}, \mathsf{E}_{\rm s} \mid r_{\rm ub} = x) f_{r_{\rm ub}}(x) {\rm d}x$. Using $f_{r_{\rm ub}}(x)$ given in (1), we get the following Proposition.

**Proposition 1.** *The joint probability of $\mathsf{E}_{\rm b}$ and $\mathsf{E}_{\rm s}$ is given by*

$$\mathcal{B}_{\rm u} =1 - 2\lambda_{\rm b}\left[\frac{\exp\left(-r_{\rm s}\left(\frac{\lambda_{\rm v}h_{\rm v}}{h_{\rm b}}+2\lambda_{\rm b}\right)\right)-\exp\left(-\frac{\lambda_{\rm v}h_{\rm v}r_{\rm s}}{h_{\rm s}}\right)}{2\lambda_{\rm b}+\lambda_{\rm v}h_{\rm v}\left(\frac{1}{h_{\rm b}}-\frac{1}{h_{\rm s}}\right)}\right.$$
$$+\frac{1-\exp\left(-r_{\rm s}\left(\frac{\lambda_{\rm v}h_{\rm v}}{h_{\rm b}}+2\lambda_{\rm b}\right)\right)}{\frac{\lambda_{\rm v}h_{\rm v}}{h_{\rm b}}+2\lambda_{\rm b}}+\frac{\exp(-2\lambda_{\rm b}r_{\rm s})}{\frac{\lambda_{\rm v}h_{\rm v}}{h_{\rm s}}+2\lambda_{\rm b}}$$
$$\left.+\frac{\exp\left(-2\lambda_{\rm b}r_{\rm s}\right)-\exp\left(-\frac{\lambda_{\rm v}h_{\rm v}r_{\rm s}}{h_{\rm s}}\right)}{\frac{\lambda_{\rm v}h_{\rm v}}{h_{\rm s}}-2\lambda_{\rm b}}\right]. \tag{5}$$

**Proposition 2.** *$\mathcal{B}_{\rm u}$ can be bounded as $\frac{e^{-2r_{\rm s}\lambda_{\rm b}}}{1+\frac{2\lambda_{\rm b}}{\lambda_{\rm v}}\frac{h_{\rm b}}{h_{\rm v}}} \leq \mathcal{B}_{\rm u} \leq \left(1-e^{-2r_{\rm s}\lambda_{\rm b}}\right)+\frac{e^{-2r_{\rm s}\lambda_{\rm b}}}{1+\frac{2\lambda_{\rm b}}{\lambda_{\rm v}}\frac{h_{\rm b}}{h_{\rm v}}}$.*

**Remark 2.** *The bounds in Proposition 2 become strict when $r_{\rm s}$ is small, i.e., $r_{\rm s} \ll 1/\lambda_{\rm b}$.*

## IV. Coverage Probability

In this section, we will compute the coverage probability of the typical user for threshold $\gamma$, which is defined as the probability that the SNR of the user is greater than $\gamma$.

**Theorem 1.** *The coverage probability of the typical general user is*

$$\mathcal{P}_{\rm u}(\gamma) = \mathbb{E}\left[F_{n_0}\left(\frac{\gamma N_0\left(r_{\rm ub}^2+h_{\rm b}^2\right)^{\frac{\alpha}{2}}}{P_{\rm t}\beta}\right)\exp\left(-\lambda_{\rm v}r_{\rm ub}\frac{h_{\rm v}}{h_{\rm b}}\right)+\right.$$
$$\left.\mathbb{P}(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid r_{\rm ub})F_{n_0}\left(\frac{\gamma N_0\left(\left((r_{\rm ub}-r_{\rm s})^2+h_{\rm s}^2\right)d_{\rm sb}^2\right)^{\frac{\alpha}{2}}}{P_{\rm t}\beta N^2\cos^2(\theta)}\right)\right],$$

*where $F_n(x) = e^{-x}\sum_{q=0}^{n-1} x^q/q!$ and the expectation is with respect to $r_{\rm ub}$ with the PDF given in (1).*

*Proof:* Let us first characterize the joint event that the user observes an LOS link to the RIS (i.e., $\bar{\mathsf{E}}_{\rm s}$) and that the link to the BS is blocked (i.e., $\mathsf{E}_{\rm b}$), given that the user-BS horizontal distance is $r_{\rm bu}$. Note that $\mathbb{P}\left(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid d_{\rm bu}\right) = \mathbb{P}\left(\mathsf{E}_{\rm b}\mid r_{\rm ub}\right) - \mathbb{P}\left(\mathsf{E}_{\rm s},\mathsf{E}_{\rm b}\mid d_{\rm bu}\right)$. Therefore, the coverage probability is

$$\mathcal{P}_{\rm u}(\gamma) =\mathbb{E}_{r_{\rm ub}}\left[\mathbb{P}(S\geq\gamma\mid r_{\rm ub})\ \mathbb{P}(\bar{\mathsf{E}}_B\mid r_{\rm ub})\right.$$
$$\left.+\mathbb{P}(S'\geq\gamma\mid r_{\rm ub})\ \mathbb{P}(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid r_{\rm ub})\right].$$

Now substituting the values of SNRs $S$ and $S'$, we get

$$\mathcal{P}_{\rm u}(\gamma) = \mathbb{E}\left[\mathbb{P}\left(\frac{P_{\rm t}}{N_0}g_{\rm bu}\beta\left(r_{\rm ub}^2+h_{\rm b}^2\right)^{-\frac{\alpha}{2}}\geq\gamma\mid r_{\rm ub}\right)\right.$$
$$\times\exp\left(-\lambda_{\rm v}r_{\rm ub}h_{\rm v}/h_{\rm b}\right)+\mathbb{P}(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid r_{\rm ub})$$
$$\left.\times\mathbb{P}\left(\frac{P_{\rm t}}{N_0}\beta d_{\rm us}^{-\alpha}d_{\rm sb}^{\alpha}\cos^2(\theta)N^2g_{\rm su}\geq\gamma\mid r_{\rm ub}\right)\right].$$

Averaging over the fading random variables $g_{\rm bu}$ and $g_{\rm su}$, we get the result. ■

**Corollary 1.** *The coverage probability of the typical general user can be approximated as*

$$\mathcal{P}_{\rm u}(\gamma)\approx \sum_{n=1}^{n_0}(-1)^{n+1}\binom{n_0}{n}$$
$$\mathbb{E}\left[\exp\left(-n\gamma'\left(r_{\rm ub}^2+h_{\rm b}^2\right)^{\alpha/2}\right)\exp\left(-\lambda_{\rm v}r_{\rm ub}\frac{h_{\rm v}}{h_{\rm b}}\right)\right.$$
$$\left.+\exp\left(-\frac{n\gamma'\left(d_{\rm us}d_{\rm sb}\right)^{\alpha}}{N^2\cos^2(\theta)}\right)\mathbb{P}(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid r_{\rm ub})\right], \tag{6}$$

*where $\gamma' = \eta_m\gamma(N_0/P_{\rm t})(1/\beta) = \frac{\eta_m\gamma N_0}{P_{\rm t}K}\frac{(4\pi)^2}{G_{\rm t}G_{\rm r}}$, $\eta_m = n_0(n_0!)^{-\frac{1}{n_0}}$, and the expectation is with respect to $r_{\rm ub}$.*

*Proof:* According to Alzer's lemma [10], [17], if $X$ is a Gamma random variable with parameter $n_0$, then its complimentary CDF (CCDF) can lower bounded as

$$F_{n_0} = \mathbb{P}(X\geq\gamma)\leq\sum_{n=1}^{n_0}(-1)^{n+1}\binom{n_0}{n}e^{-n\gamma\eta_m}. \tag{7}$$

Using (7) in Theorem 1, we get the desired result. ■

**Corollary 2.** *For the special case of $\alpha = 2$, the coverage probability can be given in a simpler form as*

$$\mathcal{P}_{\rm u}(\gamma)\approx\sum_{n=1}^{n_0}(-1)^{n+1}\binom{n_0}{n}$$
$$\left[\exp\left(\frac{1}{n\gamma'}\left(\lambda_{\rm b}+\frac{\lambda_{\rm v}h_{\rm v}}{2h_{\rm b}}\right)^2-n\gamma'h_{\rm b}^2\right){\rm erfc}\left(\frac{\lambda_{\rm b}+\frac{\lambda_{\rm v}h_{\rm v}}{2h_{\rm b}}}{\sqrt{n\gamma'}}\right)\frac{\lambda_{\rm b}\sqrt{\pi}}{\sqrt{n\gamma'}}\right.$$
$$\left.+\mathbb{E}\left[\exp\left(-\frac{n\gamma'd_{\rm us}^2d_{\rm sb}^2}{N^2\cos^2(\theta)}\right)\left(\mathbb{P}\left(\mathsf{E}_{\rm b}\,|\,r_{\rm ub}\right)-\mathbb{P}\left(\mathsf{E}_{\rm s},\mathsf{E}_{\rm b}\,|\,r_{\rm ub}\right)\right)\right]\right],$$

*where the expectation is with respect to $r_{\rm ub}$.*

*Proof:* Substituting the value of $\mathbb{P}(\bar{\mathsf{E}}_{\rm s},\mathsf{E}_{\rm b}\mid r_{\rm ub})$ using (4) and (5) and taking the expectation with respect to the distribution of $r_{\rm ub}$ as given in (1) completes the proof. ■

In the absence of blockages when the user to BS link is never blocked, we get the following result.

**Corollary 3.** *For the special case of $\alpha = 2$ and with the absence of blockages, the coverage probability $\mathcal{P}_{\rm u} = \mathbb{P}\left(S\geq\gamma\right)$ is $\lambda_{\rm b}\sqrt{\pi}\sum_{n=1}^{n_0}\binom{n_0}{n}\frac{(-1)^{n+1}}{\sqrt{n\gamma'}}\exp\left(\frac{\lambda_{\rm b}^2}{n\gamma'}-n\gamma'h_{\rm b}^2\right){\rm erfc}\left(\frac{\lambda_{\rm b}}{\sqrt{n\gamma'}}\right)$.*

## V. Impact of Intersection Mounting of RISs

In an urban setting, intersections of streets are crucial. We now consider an additional set of RISs that are mounted at each street intersection and are associated with a nearby BS. The density of the intersection-mounted RISs in the network is governed by $\lambda_{\rm r}$. We now consider an additional type of users known as the typical intersection user located at the Poisson line tessellation (PLT) crossings [18]. This is in contrast with the typical general user that can be located anywhere on the street. The typical general user connects to the nearest BS on the same street, whereas, the typical intersection user connects to the nearest BS on either of the streets that comprise the intersection. Thus, for the typical intersection user, the PDF of the horizontal distance to its serving BS is given by

$$f_{r_{\rm ib}}(x) = 4\lambda_{\rm b}\exp(-4\lambda_{\rm b}x), x \geq 0. \tag{8}$$

Equation (8) follows from the fact that, for an intersection user, there are four possible directions to the serving BS. Similar to a general user, a BS serves an intersection user by the direct link if the link is in the LOS state and from an associated RIS if the direct link is blocked (i.e., it is in the NLOS state).

We now study the impact of intersection-mounted RISs on the performance of the typical general and intersection users.

**Typical general user:** If the direct link and the associated RIS link are blocked to the typical general user, it attempts to connect to a BS from the neighboring street via the nearest intersection-mounted RIS on the opposite side as compared to the serving BS. Let the distance to the nearest intersection from the typical general user on the opposite side as the serving BS be given by $r_{\rm ux}$. The PDF of $r_{\rm ux}$ is [15]

$$f_{r_{\rm ux}}(r) = 2\lambda_{\rm r}\exp(-2\lambda_{\rm r}r). \tag{9}$$

Thus, the distance to the nearest intersection-mounted RIS is $d_{\rm ux} = \left(r_{\rm ux}^2 + h_{\rm s}^2\right)^{1/2}$. The distance between the nearest intersection-mounted RIS and the nearest BS on the adjoining street is $d_{\rm xb} = \left((h_{\rm b} - h_{\rm s})^2 + r_{\rm xb}^2\right)^{1/2}$, where $r_{\rm xb}$ is the horizontal distance of the adjoining-street nearest BS from the intersection. Note that $r_{\rm xb}$ is independent and identically distributed as $r_{\rm ub}$ and its PDF is given by (1).

Accordingly, the probability that the user is able to establish an LOS connection to the nearest intersection-mounted RIS and the link to the nearest BS as well as its associated RIS is blocked is given by the following lemma.

**Lemma 3.** *Given $r_{\rm ux}$ and $r_{\rm ub}$, the joint probability that the typical general user has an LOS link to the nearest intersection-mounted RIS while its link to the nearest BS and its associated RIS is blocked, is*

$$\mathbb{P}(\bar{\mathsf{E}}_{\rm x}, \mathsf{E}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ub}, r_{\rm ux}) =
\begin{cases}
\exp\left(-\lambda_{\rm v}\frac{h_{\rm v}}{h_{\rm s}}r_{\rm ux}\right)\left(1-\exp\left(-\lambda_{\rm v}\frac{h_{\rm v}}{h_{\rm b}}r_{\rm ub}\right)\right) & \\
\times\left(1-\exp\left(-\lambda_{\rm v}(r_{\rm s}-r_{\rm ub})\frac{h_{\rm v}}{h_{\rm s}}\right)\right) & \text{if } 0 \leq r_{\rm ub} < r_{\rm s} \\
\exp\left(-\lambda_{\rm v}\frac{h_{\rm v}}{h_{\rm s}}r_{\rm ux}\right)\left(1-\exp\left(-\lambda_{\rm v}(r_{\rm ub}-r_{\rm s})\frac{h_{\rm v}}{h_{\rm s}}\right)\right) & \\
& \text{if } r_{\rm ub} \geq r_{\rm s},
\end{cases}$$

*where $\bar{\mathsf{E}}_{\rm x}$ denotes the event that there exists an LOS path from the user to the nearest intersection-mounted RIS.*

*Proof:* The proof follows directly from the independence of the events $\bar{\mathsf{E}}_{\rm x}$, $\mathsf{E}_{\rm s}$, and $\mathsf{E}_{\rm b}$ for a given $r_{\rm ux}$ and $r_{\rm ub}$. ∎

Consequently, the coverage probability of the typical general user can be written as follows:

**Theorem 2.** *The coverage probability of the typical general user when it is aided by an intersection-mounted RIS in case of joint blockage of the serving BS and its associated RIS is*

$$\begin{aligned}
\mathcal{P}_{\rm ux} \approx \sum_{n=1}^{n_0}(-1)^{n+1}\binom{n_0}{n} & \\
\mathbb{E}\Bigg[ & \exp\left(-n\gamma'\left(r_{\rm ub}^2+h_{\rm b}^2\right)^{\frac{\alpha}{2}}\right)\exp\left(-\lambda_{\rm v}r_{\rm ub}\frac{h_{\rm v}}{h_{\rm b}}\right) \\
& + \exp\left(-\frac{n\gamma'\left(d_{\rm us}d_{\rm sb}\right)^{\alpha}}{N^2\cos^2(\theta)}\right)\mathbb{P}(\bar{\mathsf{E}}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ub}) \\
& + \exp\left(-\frac{n\gamma'\left(d_{\rm ux}d_{\rm xb}\right)^{\alpha}}{N^2\cos^2(\theta')}\right)\mathbb{P}(\bar{\mathsf{E}}_{\rm x}, \mathsf{E}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ub}, r_{\rm ux})\Bigg].
\end{aligned}$$

*Here, the expectation is taken with respect to the independent random variables $r_{\rm ub}$, $r_{\rm ux}$, and $r_{\rm xb}$. Furthermore, $\theta' = \arctan\left((h_{\rm s}-h_{\rm b})/r_{\rm xb}\right)$.*

**Typical intersection user:** The typical intersection user receives services only from the associated BS either from its corresponding RIS or the intersection-mounted RIS. If both the direct link and the associated RIS link are blocked to the typical intersection user, the BS attempts to serve the typical intersection user via the intersection-mounted RIS. Note that the distance between the typical intersection user and the base of the intersection-mounted RIS is zero ($r_{\rm ix} = 0$) since this RIS is located directly overhead the user. Thus, for the typical intersection user, the link to the nearest intersection-mounted RIS is always in LOS. Thus, for the typical intersection user, we have $\mathbb{P}(\bar{\mathsf{E}}_{\rm x}, \mathsf{E}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ib}, r_{\rm ix}) = \mathbb{P}(\mathsf{E}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ib}, r_{\rm ix})$.

**Theorem 3.** *The coverage probability of the typical intersection user aided by an intersection-mounted RIS in case of the joint blockage of the serving BS and associated RIS is*

$$\begin{aligned}
\mathcal{P}_{\rm ix} \approx \sum_{n=1}^{n_0}(-1)^{n+1}\binom{n_0}{n} & \\
\mathbb{E}\Bigg[ & \exp\left(-n\gamma'\left(r_{\rm ib}^2+h_{\rm b}^2\right)^{\frac{\alpha}{2}}\right)\exp\left(-\lambda_{\rm v}r_{\rm ib}\frac{h_{\rm v}}{h_{\rm b}}\right) \\
& + \exp\left(-\frac{n\gamma'\left(d_{\rm is}d_{\rm sb}\right)^{\alpha}}{N^2\cos^2(\theta)}\right)\mathbb{P}(\bar{\mathsf{E}}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ib}) \\
& + \exp\left(-\frac{n\gamma'\left(h_{\rm s}\sqrt{r_{\rm ib}^2+(h_{\rm b}-h_{\rm s})^2}\right)^{\alpha}}{N^2\cos^2(\theta_{\rm i})}\right)\mathbb{P}(\mathsf{E}_{\rm s}, \mathsf{E}_{\rm b} \mid r_{\rm ib})\Bigg],
\end{aligned}$$

*where $\theta_{\rm i}$ is the incidence angle at the intersection-mounted RIS from the serving BS. The expectation is taken with respect to $r_{\rm ib}$ as given in (8).*

## VI. Numerical Results and Discussions

For the evaluation purpose, the transmit power of a BS is 30 dBm, and the total beamforming gain $G_{\rm t}G_{\rm r}$ is 40 dBi.

We consider the mm-wave band of $f_c = 28$ GHz with 1 GHz bandwidth. The propagation parameters follow the 3GPP specifications as defined in [19], where $K = \left(\frac{c}{4\pi f_c}\right)^2$, the path-loss exponent $\alpha = 2$, and $n_0 = 3$. $c$ is the speed of light in free space. The number of elements in each RIS is $N = 100$. The noise power spectral density is $-174$ dBm/Hz. Unless mentioned explicitly, we assume $\lambda_{\rm b} = 0.1$ /m, $h_{\rm b} = 10$ m, $h_{\rm r} = 50$ m, $\lambda_{\rm v} = 0.3$ /m and $h_{\rm v} = 3$ m.

**Blockage probability.** Fig. 2 shows that the probability that the links from the typical general user to its serving BS and the associated RISs are jointly blocked, is minimized for a certain value $r_{\rm s,o}$ of $r_{\rm s}$ (i.e., the distance of the RIS from its coupled BS). Let $R = 1/\lambda_{\rm b}$ denote the order of average cell radius. As seen from Remark 1, if the RISs are deployed very far from most users (i.e., $r_{\rm s} \gg R$) or very close to BSs (i.e. $r_{\rm s} \approx 0$), $\mathcal{B}_{\rm u}$ is high. This optimal value of $r_{\rm s}$ is of the order of $R$ and depends on the blockage density $\lambda_{\rm v}$ and the blockage height $h_{\rm v}$. When blockages are sparse (i.e., $\lambda_{\rm v}$ and $h_{\rm v}$ are small), cell edge users (i.e., $r_{\rm ub} \approx R$) still have a chance to be blocked. Hence, they determine the optimal value of $r_{\rm s}$ to be equal to their location, i.e., $r_{\rm s,o} \approx R$. On the other hand, in the presence of severe blockages, even the users near BS (i.e., $r_{\rm ub} \approx R$) can be blocked, hence the $r_{\rm s,o}$ shifts towards $R/2$. One can also observe that the lower bound (given by Proposition 2) is tight for small values of $r_{\rm s}$ since it is obtained by considering only the case when $r_{\rm ub} \geq r_{\rm s}$.

**Coverage probability.** Fig. 3 shows that the coverage probability also attains a maximum value for a certain optimal value $r'_{\rm s,o}$ of $r_{\rm s}$. As the deployment of the BSs gets denser, the optimal distance of the RIS to BS decreases and remains of the order of $R$. For example, for $\lambda_{\rm b} = 0.05$ m$^{-1}$, $R = 20$ m and $r'_{\rm s,o} = 12.5$ m, whereas it decreases to 5 m for $\lambda_{\rm b} = 0.02$ m$^{-1}$ ($R = 5$ m). Due to the high probability of joint blockage (as shown in Fig. 2), the coverage probability decreases for sufficiently low and high values of $r_{\rm s}$.

Interestingly, Fig. 4 shows that for a BS height of 10 m, the optimal height for deploying the RIS is about 50 m, regardless of the height and the density of the blockages. A further increase in $h_{\rm s}$ reduces the coverage probability $\mathcal{P}_{\rm u}$. For very large values of $h_{\rm s}$, $\mathcal{P}_{\rm u}$ becomes constant because the coverage is only due to the direct link to the BS owing to the large distance between the RIS and the user.

**Impact of intersection-mounted RIS.** Fig. 5 shows that leveraging an intersection-mounted RIS when the serving BS and its associated RIS are jointly blocked for the typical general user increases the coverage probability. Interestingly, the optimal height of RISs also increases in this case. For a dense road network (i.e., high $\lambda_{\rm r}$), the intersection-mounted RIS is near to the user, which enables a higher RIS deployment while still maintaining the user coverage. Also, with a higher density $\lambda_{\rm r}$ of the streets, the number of intersection-mounted-RISs increases, and hence the coverage is larger.

For users associated to a BS that is close to an intersection, an intersection-mounted RIS might provide a higher signal power via a BS on the adjoining street compared to the RIS

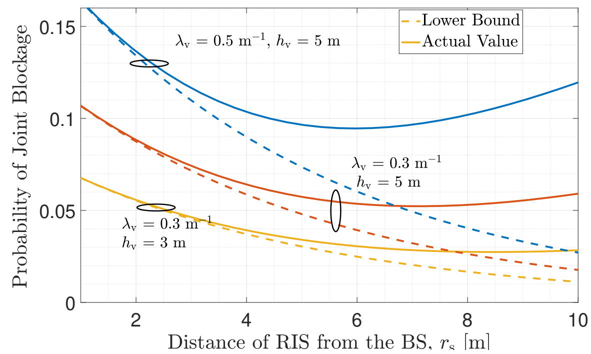


Fig. 2. Joint blockage probability $\mathcal{B}_{\rm u}$ with respect to the deployment distance of the RIS for different values of blockage density and height. Here $h_{\rm b} = 10$ m, $h_{\rm s} = 50$ m, and $\lambda_{\rm b} = 0.1$ m$^{-1}$.

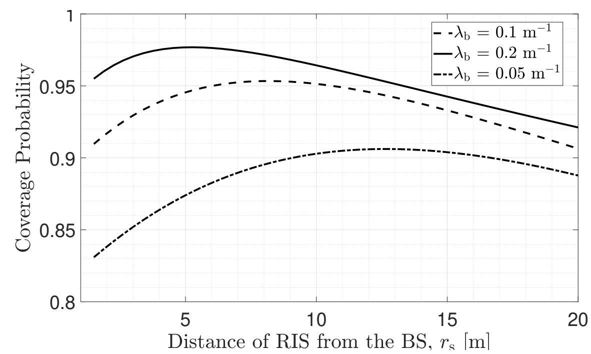


Fig. 3. Coverage probability $\mathcal{P}_{\rm u}$ as a function of the deployment distance of the RIS for different BS densities.

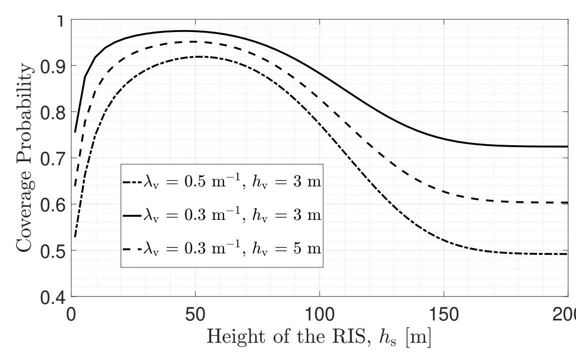


Fig. 4. Coverage probability $\mathcal{P}_{\rm u}$ with respect to the deployment height of the RIS for different blockage height and densities. Here, $\lambda_{\rm b}$ = 0.1 m$^{-1}$.

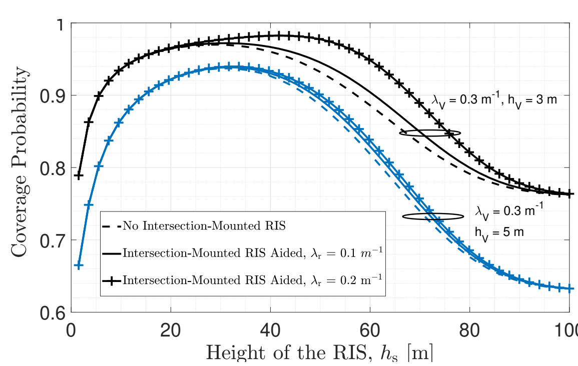


Fig. 5. Impact of the intersection-mounted RIS on the coverage probability of typical general user with respect to the deployment height of the RIS.

coupled with the serving BS.

In Fig. 6, we plot the outage probability of a tagged user uniformly located in the coverage area of a BS. We tune the distance between the serving BS and its nearest intersection. The coupled RIS is deployed at an optimized distance and height for a given $\lambda_{\rm b}$ according to Fig. 3 and Fig. 4, respectively. Then the user is assumed to select either 1) the coupled RIS of the serving BS or 2) the intersection RIS serving via a BS on an adjoined street, based on higher downlink power. For BSs deployed very close to the intersection (e.g., $< 50$ m with $\lambda = 0.05$ m$^{-1}$), a higher coverage probability is achieved when the user always connects to the intersection-mounted RIS. On the contrary, for higher BS densities (e.g., $\lambda_{\rm b} = 0.1$ m$^{-1}$), the coupled RIS is deployed nearer to the serving BS and thus provides a better coverage. Consequently, based on the deployment density of the BSs, a network operator may avoid deploying RISs for BSs located very close to the intersections.

Fig. 7 shows that, since the link between the intersection user and an intersection-mounted RIS is always in the LOS state, such a user benefits from a higher mounting of the RIS. This increases the LOS probability for the link between the serving BS and the associated RIS.

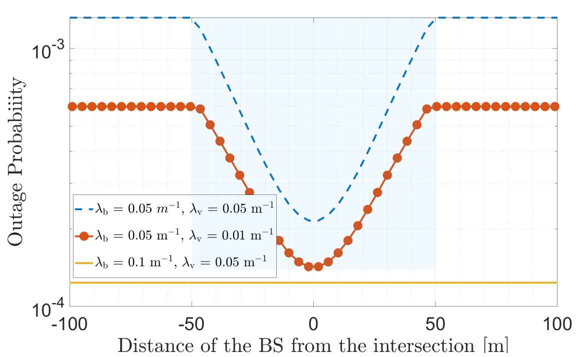


Fig. 6. Outage probability of a user that selects either the intersection RIS or the associated RIS based on the higher received power. $\lambda_{\rm r}$ = 0.1 m$^{-1}$.

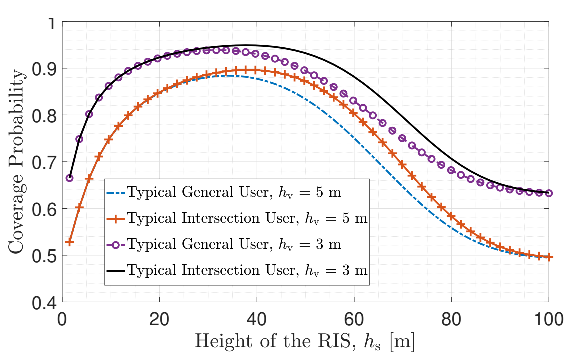


Fig. 7. Coverage probabilities of the typical general and intersection users against the deployment height of the RIS. $\lambda_{\rm r}$ = 0.1 m$^{-1}$.

## VII. Conclusions

Deploying RISs very close or very far from the BS increases the probability that the links from the user to the BS and to the RIS are simultaneously blocked. This results in worse coverage for users. In this paper, we studied the optimal placement of RISs from the perspective of the coverage. We showed that there is an optimal location inside the coverage region that improves the coverage for users on an average. We also observed that leveraging intersection-mounted RISs improves the coverage. Also, the typical intersection user enjoys a higher coverage probability compared to the typical general user due to the existence of an LOS link from the former to the intersection-mounted RIS. Interestingly, for users associated to BSs that are deployed very close to an intersection, selecting an intersection-mounted RIS provides a higher coverage compared to selecting an RIS associated with the serving BS.

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