On the Performance of Generalized Energy Detector Under Noise Uncertainty in Cognitive Radio

Sanket S. Kalamkar and Adrish Banerjee
Department of Electrical Engineering
Indian Institute of Technology
Kanpur 208016, India
Email: {kalamkar, adrish}@iitk.ac.in

***Abstract*—In cognitive radio, spectrum sensing is a fundamental task and is used to detect primary user. Energy detection is a popular spectrum sensing technique. But detection performance of energy detector (ED) deteriorates in low signal-to-noise ratio (SNR) conditions and under noise uncertainty. In this paper, we study generalized energy detector (GED), obtained by replacing squaring operation of amplitude of the received signal in conventional energy detector (CED) with an arbitrary positive power operation p under noise uncertainty. For the worst case of noise uncertainty we analytically show that SNR wall is not dependent on the value of p. We further investigate the detection performance of GED for different values of p under uniformly distributed noise uncertainty and show that CED is the best ED under noise uncertainty. We also show that at noise uncertainty greater than 0.5 dB, the performance gap between different EDs almost vanishes and the detection performances of all EDs almost become the same for all values of p.**

I. Introduction

Cognitive radio [1], [2] is an exciting emerging paradigm which may be considered as a solution to inefficient usage [3] of fixed allocated licensed frequency spectrum. Significant improvement in spectrum utilization can be achieved by allowing an unlicensed or secondary user (SU) to access a licensed frequency band when the licensed or primary user (PU) is absent. In cognitive radio, SU senses an idle frequency band of a PU, and if a band is found to be idle, SU may transmit over that band. But as soon as PU returns, SU must vacate the band immediately. This complete process requires accurate spectrum sensing to avoid harmful interference to PU.

There are quite a few sensing methods that have been proposed, like likelihood ratio test [4], matched filtering based detection [4], cyclostationary detection [5], covariance based detection [6], eigenvalue based detection [7] and energy detection (ED) [4], [8], [9]. ED requires no a priori knowledge about primary signals. Also ED is less complex and easy to implement than the other spectrum sensing techniques. Conventional energy detector (CED) [8] can be generalized by replacing squaring operation of received signal amplitude by an arbitrary positive power operation constant p [10]. We call this modified ED as generalized energy detector (GED). Then CED becomes a special case of GED with $p = 2$. We briefly review CED and GED in section II. In [10], [11], [12], it has been shown that detection performance of SU may be improved by choosing a suitable value for p and this chosen value of p may not be equal to 2 i.e. CED may not be the best ED.

978-1-4673-5952-8/13/$31.00

In GED, the test statistic is compared with a predetermined threshold to take a decision on presence or absence of PU. To calculate threshold accurately one needs exact knowledge of noise power/variance. With noise power known precisely, theoretically it is possible for ED to detect the presence of PU even at very low SNR if the sensing time is made very large. But in practice, noise power may change with time and location. Therefore it may not be possible to measure exact noise power at a particular time and location. In [13], [14], the effect of worst case noise uncertainty on CED has been studied in detail. The worst case of noise uncertainty is considered assuming that the noise power is constrained to the limited range where only upper and lower bound on noise variance are known to the detector. In [14], it has been proved by the authors that if noise variance is not known exactly and confined to an interval, the phenomenon known as *SNR wall* exists, for which targeted detection performance cannot be achieved regardless of the sensing time and ED does not remain an effective detection method under noise uncertainty.

In [15], authors have analysed the performance of CED under the assumption that noise uncertainty is uniformly distributed. In [16], discrete and continuous forms of the noise uncertainty model are proposed and it is shown that choosing different statistical decision threshold results in different detection performance. Log-Normal approximated noise uncertainty is assumed for CED in [17]. In [18], authors perform asymptotic analysis of noise power estimation for CED. The conditions for the existence of SNR wall are derived and effect of noise power estimation on detection performance of CED is studied.

In this paper, we primarily build on [10], [14] and [15]. We study the performance of GED under noise uncertainty. We analytically show that for the worst case of noise uncertainty described in [14], SNR wall is independent of power constant p. The performance of GED is also shown for uniform distribution of noise uncertainty [15]. We also show that CED (p = 2) is the best ED under noise uncertainty. But when the noise uncertainty is absent, the best ED may be different than CED. We further numerically show that if the noise uncertainty is significant (greater than 0.5 dB), then there is a very little effect of p on the detection performance of GED.

The rest of the paper is organized as follows. In section II, we describe the system model and briefly review the concepts of conventional energy detection and its extension to generalized energy detection. Section III has been divided into two parts. The first part shows analytically that for the worst case of noise uncertainty considered in [14] for GED, SNR wall remains the same irrespective of value of power constant p. In the second part, uniform distribution of noise uncertainty is considered for GED. We provide expressions of average probability of detection and average probability of false alarm which can be evaluated numerically. In section IV, we provide numerical results. We evaluate the detection performance for various system parameters like power constant p, noise uncertainty and SNR. Finally conclusions are drawn in section V.

II. Background

In cognitive radio, PU detection is a binary hypothesis problem in classical detection theory [4] which is given as

$$y(n) = \begin{cases} hs(n) + w(n), & H_1 \\ w(n), & H_0 \end{cases} \quad (1)$$

where $n = 1, \ldots, N$ indexes the samples of received signal by SU, $y(n)$ is the nth received signal sample by SU, $s(n)$ is the nth unknown primary signal sample, h represents fading channel coefficients of the propagation channel between PU and SU and $w(n)$ is additive white Gaussian noise (AWGN) with mean zero and variance σ^2. H_1 and H_0 are the hypotheses corresponding to presence and absence of PU respectively. The average power of primary signal is σ_s^2. We assume that primary signal is independent of noise and fading. It is considered that primary signal samples are independent. Noise samples are also assumed to be independent. For simplicity, we consider primary signal, fading coefficients and noise are real number. Extension of the results for complex signals can easily be done.

The aim of spectrum sensing is to decide the presence or absence of PU based on aforementioned binary hypothesis problem (choose H_1 or H_0). The decision is taken based on received signal by the secondary user. Spectrum sensing algorithm performance is generally measured by two probabilities: Probability of detection (P_D) and probability of false alarm (P_{FA}) which are defined as

$$P_D = Pr(H_1|H_1) \quad (2)$$

$$P_{FA} = Pr(H_1|H_0) \quad (3)$$

Thus the probability of detection is the probability of choosing H_1 when the true hypothesis is H_1 and probability of false is the probability of choosing H_1 when the true hypothesis is H_0. A good sensing algorithm is the one that achieves high probability of detection and low probability of false alarm, for a given number of samples.

In this paper, the detection method used for spectrum sensing is energy detection since it does not require prior knowledge of primary signals and is easy to implement because of low complexity. In conventional energy detector (CED), the received signal samples are first squared, then summed over the number of samples collected and then compared with a predetermined threshold to take decision on presence or absence of PU. The test statistic T_{CED} for conventional energy detector is given as

$$T_{CED} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^2 \quad (4)$$

where N is the number of samples.

We can transform conventional energy detector to generalized energy detector [10] by replacing squaring operation by an arbitrary positive operation p. Then the test statistic for GED is given as

$$T_{GED} = \frac{1}{N}\sum_{n=1}^{N} |y(n)|^p \quad (5)$$

where $p > 0$ is an arbitrary constant. It can be seen that CED is a special case of GED with $p = 2$.

For large N and thus invoking central limit theorem (CLT) [15], we can define probability of detection P_D and probability of false alarm P_{FA} for GED as

$$P_D = Pr(T_{GED} > T|H_1) = Q\left(\frac{T - \mu_1}{\sigma_1/\sqrt{N}}\right) \quad (6)$$

and

$$P_{FA} = Pr(T_{GED} > T|H_0) = Q\left(\frac{T - \mu_0}{\sigma_0/\sqrt{N}}\right) \quad (7)$$

where

$$Q(t) = \frac{1}{\sqrt{2\pi}}\int_t^{\infty} e^{-(x^2/2)}dx \quad (8)$$

and T is the predetermined threshold which can be obtained by fixing probability of false alarm, μ_1 and μ_0 are means of T_{GED} under H_1 and H_0 respectively, σ_1^2 and σ_0^2 are variances of T_{GED} under H_1 and H_0 respectively, which can be given as [10]

$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma^p \quad (9)$$

$$\sigma_0^2 = \frac{2^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma^{2p} \quad (10)$$

$$\mu_1 = \frac{2^{p/2}(1+\gamma)^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma^p \quad (11)$$

$$\sigma_1^2 = \frac{2^p(1+\gamma)^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma^{2p} \quad (12)$$

with γ is average received signal-to-noise ratio (ASNR).

III. Noise Uncertainty Model for Generalized Energy Detector

From (6), (7), (9), (10), (11) and (12), it can be seen that P_D and P_{FA} depend on the threshold T and noise variance σ^2, and to set the threshold one needs exact knowledge of noise power. In general it is assumed that noise power is known a *priori*.

But in practical scenario this is not the case. Variance/power of white noise is the only parameter on which noise distribution is dependent. However, as mentioned earlier, in practice there exists noise uncertainty [14] since noise power may change with time and location and is not known exactly. The presence of noise uncertainty makes it very difficult to obtain exact noise power at a particular time and location.

A. Worst Case of Noise Uncertainty

In this section, we consider the worst case of noise uncertainty proposed in [14] where only upper and lower bound on noise uncertainty are known.

Definition 1 (SNR wall): SNR wall is the minimum SNR below which spectrum sensing cannot be performed reliably i.e. probability of detection becomes smaller than 0.5 and/or probability of false alarm becomes greater than 0.5 [14].

We state a theorem as

Theorem 1. *If only upper and lower bound on noise uncertainty are known for GED, then SNR wall is independent of value of p.*

Proof: We assume that uncertainty in noise power is distributed in a single interval $\sigma^2 \in \left[(1/\rho)\hat{\sigma}_w^2, \rho\hat{\sigma}_w^2\right]$ as mentioned in [14] where $\hat{\sigma}_w^2$ is the expected/average noise power and $\rho > 1$ is a noise uncertainty parameter. Then P_D and P_{FA} can be written as

$$P_D = \min_{\sigma^2 \in [(1/\rho)\hat{\sigma}_w^2, \rho\hat{\sigma}_w^2]} Q\left(\frac{T-\mu_1}{\sigma_1/\sqrt{N}}\right) \tag{13}$$

and

$$P_{FA} = \max_{\sigma^2 \in [(1/\rho)\hat{\sigma}_w^2, \rho\hat{\sigma}_w^2]} Q\left(\frac{T-\mu_0}{\sigma_0/\sqrt{N}}\right) \tag{14}$$

where μ_0, σ_0^2, μ_1 and σ_1^2 are given by (9), (10), (11) and (12) respectively.

Eliminating T, we can write

$$N = \frac{\left[\sigma_0 Q^{-1}(P_{FA}) - \sigma_1 Q^{-1}(P_D)\right]^2}{(\mu_1-\mu_0)^2} \tag{15}$$

From above equation, it can be easily seen that $N \to \infty$ as $\mu_1 \to \mu_0$. Thus it is not possible to achieve target P_D and P_{FA} robustly if $\mu_1 \leq \mu_0$, giving rise to a phenomenon known as "SNR wall" as described below.

For the worst case of noise uncertainty, mentioned above, we can write μ_0 and μ_1 for GED using (9) and (11) as

$$\mu_0 = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\rho^{p/2}\hat{\sigma}_w^p \tag{16}$$

$$\mu_1 = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\left(\frac{1}{\rho}\right)^{p/2}\hat{\sigma}_w^p\left(1+\frac{\rho\sigma_s^2}{\hat{\sigma}_w^2}\right)^{p/2} \tag{17}$$

where σ_s^2 is average power of primary signal.

We can write the condition $\mu_1 \leq \mu_0$ as

$$\rho^{p/2} \leq \left(\frac{1}{\rho}\right)^{p/2}\left(1+\frac{\rho\sigma_s^2}{\hat{\sigma}_w^2}\right)^{p/2} \tag{18}$$

$$\Rightarrow \gamma \leq (\rho - (1/\rho)) \tag{19}$$

with $\gamma = \sigma_s^2/\hat{\sigma}_w^2$, average received SNR (ASNR). From (19), it can be seen that if average SNR is less than the uncertainty in noise power, then primary signal cannot be detected reliably, which corresponds to the SNR wall. The SNR wall for GED shown in (19) is exactly the same as the SNR wall for CED shown in [14]. Thus we can claim that for the worst case of noise uncertainty, SNR wall is independent of p and there is no performance change in terms of SNR wall with p. ■

B. Noise Uncertainty With Uniform Distribution

In this section, we build up on [15] by analysing performance of generalized energy detectors under uniformly distributed noise uncertainty.

In practice, the average noise power is known. Let the average noise power be $\hat{\sigma}_w^2$. At a fixed time and location, let the actual noise power be σ_w^2 which may be different from than that of the average noise power $\hat{\sigma}_w^2$, which gives rise to the noise uncertainty. So we can define the noise uncertainty factor as $\beta = \frac{\hat{\sigma}_w^2}{\sigma_w^2}$. Let the upper bound on noise uncertainty factor in dB be L which is defined as

$$L = \sup\{10\log_{10}\beta\} \tag{20}$$

We assume that noise uncertainty factor β in dB is uniformly distributed in the range [-L, L] [14]. This means β is restricted in $[10^{-L/10}, 10^{L/10}]$. In practice normally the upper bound on noise uncertainty is below 2 dB. Now as β (in dB) i.e. $10\log_{10}\beta$ is uniformly distributed in [-L, L], the probability density function (pdf) of β can be given by using simple transformation of random variable as

$$f_\beta(x) = \begin{cases} 0, & x < 10^{-L/10} \\ \dfrac{5}{\ln(10)Lx}, & 10^{-L/10} < x < 10^{L/10} \\ 0, & x > 10^{L/10} \end{cases} \tag{21}$$

where $\ln(z)$ is natural logarithm of z.

Let $k\hat{\sigma}_w^2$ be the threshold for GED, where k is constant and $\hat{\sigma}_w^2$ is average noise power as defined earlier. Thus actual noise power is $\sigma_w^2 = \frac{\hat{\sigma}_w^2}{\beta}$. Also $\gamma = \frac{\sigma_s^2}{\hat{\sigma}_w^2}$ is the average received SNR. For this set-up, means ($\mu_{0,nu}$, $\mu_{1,nu}$) and variances ($\sigma_{0,nu}^2$, $\sigma_{1,nu}^2$) under H_0 and H_1 for T_{GED} can be obtained from (9)-(12) by replacing σ with σ_w and adding noise uncertainty factor β, and are given as

$$\mu_{0,nu} = \frac{2^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma_w^p \tag{22}$$

$$\sigma_{0,nu}^2 = \frac{2^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma_w^{2p} \tag{23}$$

$$\mu_{1,nu} = \frac{2^{p/2}(1+\beta\gamma)^{p/2}}{\sqrt{\pi}}\Gamma\left(\frac{p+1}{2}\right)\sigma_w^p \tag{24}$$

$$\sigma_{1,nu}^2 = \frac{2^p(1+\beta\gamma)^p}{\sqrt{\pi}}\left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}}\Gamma^2\left(\frac{p+1}{2}\right)\right]\sigma_w^{2p} \tag{25}$$

Let us define

$$G_p = \frac{2^{p/2}}{\sqrt{\pi}} \Gamma\left(\frac{p+1}{2}\right) \quad (26)$$

and

$$K_p = \frac{2^p}{\sqrt{\pi}} \left[\Gamma\left(\frac{2p+1}{2}\right) - \frac{1}{\sqrt{\pi}} \Gamma^2\left(\frac{p+1}{2}\right) \right] \quad (27)$$

Then the probability of detection P_D and probability of false alarm P_{FA} for fixed β can be given from (6), (7) and (22)-(27) as

$$\begin{aligned} P_D &= Pr(T_{GED} > k\hat{\sigma}_w^2 | H_1) \\ &= Q\left(\left(\frac{k\beta^{p/2} - G_p(1+\beta\gamma)^{p/2}}{(1+\beta\gamma)^{p/2}} \right) \sqrt{\frac{N}{K_p}} \right) \end{aligned} \quad (28)$$

and

$$\begin{aligned} P_{FA} &= Pr(T_{GED} > k\hat{\sigma}_w^2 | H_0) \\ &= Q\left(\left(k\beta^{p/2} - G_p \right) \sqrt{\frac{N}{K_p}} \right) \end{aligned} \quad (29)$$

Now we can get average probability of detection $\bar{P}_D$ and average probability of false alarm $\bar{P}_{FA}$ by averaging over noise uncertainty factor β, using (21), (28) and (29) as

$$\begin{aligned} \bar{P}_D &= \int_{-\infty}^{\infty} Q\left(\left(\frac{kx^{p/2} - G_p(1+x\gamma)^{p/2}}{(1+x\gamma)^{p/2}} \right) \sqrt{\frac{N}{K_p}} \right) f_\beta(x) dx \\ &= \int_{10^{-L/10}}^{10^{L/10}} Q\left(\left(\frac{kx^{p/2} - G_p(1+x\gamma)^{p/2}}{(1+x\gamma)^{p/2}} \right) \sqrt{\frac{N}{K_p}} \right) \\ &\quad \frac{5}{\ln(10)Lx} dx \end{aligned} \quad (30)$$

and

$$\begin{aligned} \bar{P}_{FA} &= \int_{-\infty}^{\infty} Q\left(\left(kx^{p/2} - G_p \right) \sqrt{\frac{N}{K_p}} \right) f_\beta(x) dx \\ &= \int_{10^{-L/10}}^{10^{L/10}} Q\left(\left(kx^{p/2} - G_p \right) \sqrt{\frac{N}{K_p}} \right) \frac{5}{\ln(10)Lx} dx \end{aligned} \quad (31)$$

Since (30) and (31) cannot be obtained in closed form, we evaluate them numerically.

IV. Numerical Results and Discussion

In this section, we present our numerical results to describe the performance of generalized energy detector and effect of noise uncertainty on it.

Fig. 1 shows receiver operating characteristic (ROC) curve where average probability of detection $\bar{P}_D$ is plotted against average probability of false alarm $\bar{P}_{FA}$ for different values of p with noise uncertainty L = 0.1 dB, N = 10000 and ASNR = -15 dB. It can be seen that the best energy detector that gives rise to maximum area under ROC curve is the one with $p = 2$, that is, CED. For any values of p other than 2, the detection performance degrades compared to that of CED. This can also be verified from Fig. 2 where $\bar{P}_D$ is plotted against ASNR for fixed $\bar{P}_{FA}$. CED (p = 2) is the best energy detector among all energy detectors and the detection performance degrades as p deviates from 2.

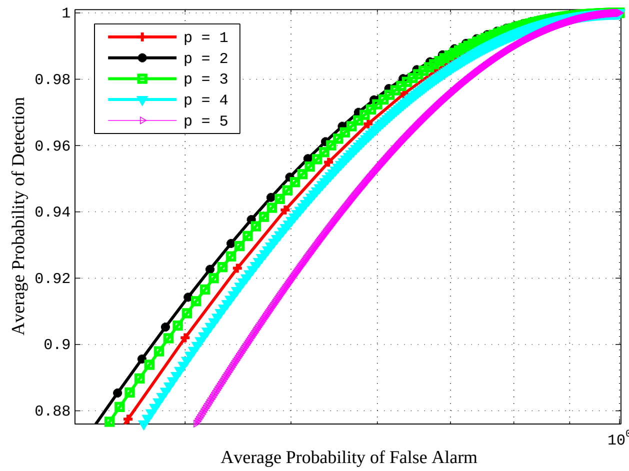

Fig. 1. ROC curve for different values of p for L = 0.1 dB, N = 10000, ASNR = -15 dB.

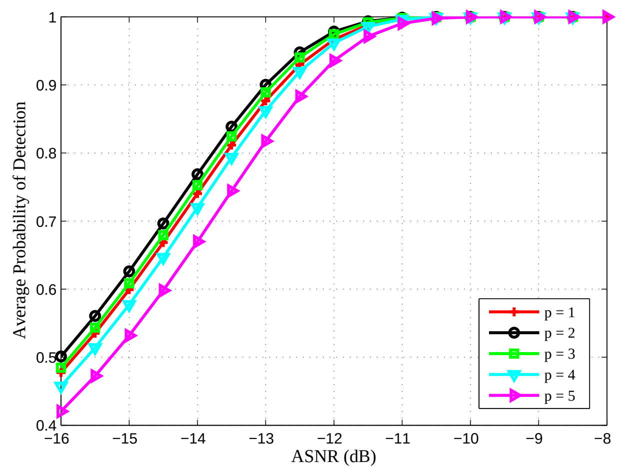

Fig. 2. $\bar{P}_D$ vs. ASNR (dB) for different values of p for L = 0.1 dB, N = 10000, $\bar{P}_{FA}$ = 0.1.

Fig. 3 compares energy detectors with $p = 2$ and $p = 5$ for the cases when there is no noise uncertainty ($L = 0$ dB), $L =$ 0.2 dB and $L = 0.5$ dB. When there is no noise uncertainty, the detection performance gap between GED with $p = 2$ and GED with $p = 5$ is large, former performing better than that of the latter. But as the noise certainty increases, the performance gap decreases. For significant noise uncertainty ($L \geq 0.5$ dB), this gap is negligible and all energy detectors perform almost the same, that is, the detection performance becomes independent of p for significantly large noise uncertainty.

Fig. 4 shows the variation of $\bar{P}_D$ versus power constant p for L = 0.1 dB, L = 0.25 dB and no noise uncertainty (L = 0 dB) with $\bar{P}_{FA}$ = 0.1, N = 10000 and ASNR = -15 dB. We consider two cases: The first, when noise uncertainty is present and the

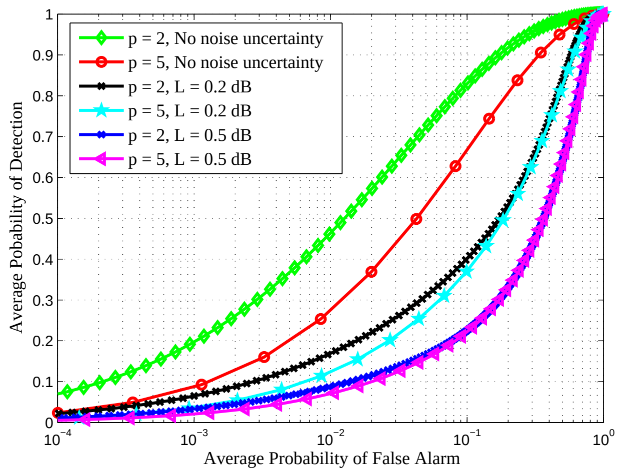

Fig. 3. Comparison of ROC curves for $p = 2, 5$ with no noise uncertainty, L = 0.2 dB and L = 0.5 dB, N = 10000, ASNR = -15 dB.

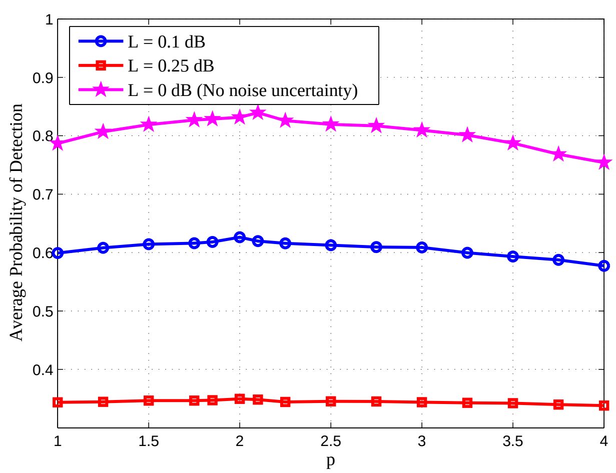

Fig. 4. $\bar{P}_D$ vs. p for $L = 0.1$ dB, 0.25 dB for $\bar{P}_{FA} = 0.1$, N = 10000, ASNR = -15 dB.

second, when there is no noise uncertainty. For the first case, from Fig. 4, it can be verified that GED with $p = 2$ is the best detector for both L = 0.1 dB and L = 0.25 dB. For L = 0.1 dB, the detection performance degrades significantly as the p deviates from 2. For $p = 2$, $\bar{P}_D$ is 0.6262 which deteriorates to 0.5773 for $p = 4$. However, for $L = 0.25$ dB, $\bar{P}_D$ deteriorates not significantly, from 0.3496 to 0.3380 as p changes from 2 to 4. This highlights the fact that more the noise uncertainty, lesser is the effect of p on the detection performance and with significantly high value of noise uncertainty, the detection performance becomes independent of p, which is also shown in Fig. 3. Also it can be observed from Fig. 4 that when there is no noise uncertainty, the best ED that has the maximum $\bar{P}_D$, corresponds to $p = 2.1$ and CED ($p = 2$) is not the best ED. But when the noise uncertainty is present, CED is the best ED .

V. Conclusion

In this paper, the detection performance of generalized energy detector is analysed, under the worst case of noise uncertainty and under the assumption that noise uncertainty is uniformly distributed. For the worst case of noise uncertainty, analytically it is shown that SNR wall remains unchanged for all values of p. Under the noise uncertainty with uniform distribution, generalized energy detector with $p = 2$ i.e. conventional energy detector, is the best energy detector. But conventional energy detector may not be the best energy detector in the absence of noise uncertainty. Also as the noise uncertainty increases and becomes significant (generally greater than 0.5 dB), the detection performance of generalized energy detector becomes independent of p.

References

[1] J. Mitola and G. Q. Maguire, "Cognitive radio: Making software more personal," *IEEE Personal Communications*, vol. 6, pp. 13–18, Aug 1999.

[2] S. Haykin, "Cognitive radio: Brain-empowered wireless communications," *IEEE Journal on Selected Areas in Communications*, vol. 23, pp. 201–220, Feb. 2005.

[3] FCC, "Spectrum policy task force," *ET Docket 02-135*, Nov. 2002.

[4] S. M. Kay, *Fundamentals of Statistical Signal Processing, Volume II: Detection Theory*. Prentice Hall, 1998.

[5] W. Gardner, "Exploitation of spectral redundancy in cyclostationary signals," *IEEE Signal Processing Magazine*, vol. 8, no. 2, pp. 14 –36, 1991.

[6] Y. Zeng and Y.-C. Liang, "Covariance based detections for cognitive radio," in *Proc. IEEE Dynamic Spectrum Access Networks (DySPAN'05)*, pp. 202–207, 2007.

[7] Y. Zeng and Y.-C. Liang, "Maximum-minimum eigenvalue detection for cognitive radio," in *Proc. IEEE International Symposium on Personal, Indoor and Mobile Radio Communication, (PIMRC'07)*, pp. 1–5, 2007.

[8] H. Urkowitz, "Energy detection of unknown deterministic signals," *Proc. IEEE*, vol. 55, pp. 523–531, Apr. 1967.

[9] F. Digham, M. Alouini, and M. Simon, "On the energy detection of unknown signals over fading channels," *IEEE Transactions on Communications*, vol. 55, pp. 21–24, Jan. 2007.

[10] Y. Chen, "Improved energy detector for random signals in Gaussian noise," *IEEE Transactions on Wireless Communications*, vol. 9, pp. 558–563, Feb. 2010.

[11] J. Song, Z. Feng, P. Zhang, and Z. Liu, "Spectrum sensing in cognitive radios based on enhanced energy detector," *IET Communications*, vol. 6, pp. 805–809, May 2012.

[12] A. Singh, M. Bhatnagar, and R. Mallik, "Cooperative spectrum sensing in multiple antenna based cognitive radio network using an improved energy detector," *IEEE Communications Letters*, vol. 16, pp. 64–67, Jan. 2012.

[13] A. Sonnenschein and P. Fishman, "Radiometric detection of spread-spectrum signals in noise of uncertain power," *IEEE Transactions on Aerospace and Electronic Systems*, vol. 28, pp. 654–660, July 1992.

[14] R. Tandra and A. Sahai, "SNR walls for signal detection," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, pp. 4–17, Feb. 2008.

[15] Y. Zeng, Y.-C. Liang, A. Hoang, and C. Peh, "Reliability of spectrum sensing under noise and interference uncertainty," in *Proc. IEEE International Conference on Communications Workshops, (ICC'09)*, pp. 1–5, 2009.

[16] W. Lin and Q. Zhang, "A design of energy detector in cognitive radio under noise uncertainty," in *Proc. IEEE International Conference on Communication Systems (ICCS'08)*, pp. 213–217, 2008.

[17] W. Jouini, "Energy detection limits under Log-Normal approximated noise uncertainty," *IEEE Signal Processing Letters*, vol. 18, pp. 423–426, July 2011.

[18] A. Mariani, A. Giorgetti, and M. Chiani, "SNR wall for energy detection with noise power estimation," in *Proc. IEEE International Conference on Communications (ICC'11)*, pp. 1–6, June 2011.